A Dynamic Factor Analysis of the Response of U. S. Interest Rates to News

Marco Lippi
Dipartimento di Scienze Economiche, Universitá di Roma 1
Daniel L. Thornton
Federal Reserve Bank of St. Louis

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Abstract

This paper uses a dynamic factor model recently studied by Forni, Hallin, Lippi and Reichlin (2000) to analyze the response of 21 U.S. interest rates to news. Using daily data, we find that the news that affects interest rates daily can be summarized by two common factors. This finding is robust to both the sample period and time aggregation. Each rate has an important idiosyncratic component; however, the relative importance of the idiosyncratic component declines as the frequency of the observations is reduced, and nearly vanishes when rates are observed at the monthly frequency. Using an identification scheme that allows for the fact that when policy actions are unknown to the market the funds rate should respond first to policy actions, we are unable to identifying a unique effect of monetary policy in the funds rate at the daily frequency.

1 Introduction

Factor analysis has been widely used in economics and finance in situations where a relatively large number of variables are believed to be driven by relatively few common causes of variation. In particular, factor analysis has been applied widely in analyzes of financial markets because alternative debt instruments are merely promises made by different economic entities to pay
various sums of money at various future dates. Economists and financial
market analysts believe that most of the variation among interest rates on
alternative debt instruments is determined by default and market risk, the
latter being positively related to the term to maturity. If financial markets are
efficient, the long-run equilibrium real return on alternative debt instruments
should differ only by default and market risk premiums.

Of course, in the short run interest rates will differ for wide variety of
causes, represented by news affecting the markets each day. However, though
the source of the news can change from day to day, the response of the interest
rates is likely to compress their informational content into a few main causes
of variation. In particular, the news will affect the interest rates by changing
the market perception about the real interest rate and expected inflation.

We investigate the common components to news by analyzing daily changes
in U.S. interest rates using the dynamic factor model (DFM) studied re-
cently by Forni, Hallin, Lippi and Reichlin (2000) (FHLR henceforth). Like
principal components (PC), DFMs identify common factors associated with
changes in interest rates. However, because the common components are
loaded through (finite or infinite) polynomials in the lag operator $L$, unlike
static PC, DFMs permit each rate to have a different dynamic response to
news. If financial markets are fully efficient, the reaction to news will be
immediate–daily changes in market interest rates will reflect completely the
information received by the market that day. If markets are not fully ef-
cient, however, the response of rates to each day’s news will evolve over
time. Hence, information about the extent of financial market efficiency is
obtained by comparing the common factors obtained with PC with those
obtained using the DFM.

The DFM is similar to the moving average version of a vector autoregres-
sion (VAR) model; however, unlike VARs, the number of common factors is
permitted to be small relative to the number of variables considered. In ad-
dition, factor analysis provides a straightforward measure of market specific,
and other idiosyncratic shocks associated with particular interest rates, and
a method of determining whether these components have a lasting affect on
that rate. In addition, it provides a measure of the relative importance of
the idiosyncratic component to that of the common factors for each rate.

Summing up, this paper attempts to answer several questions concerning
the response of interest rates to news, such as: Can the response of a variety
of interest rates to news be summarized by a few common factors? How
many common factors are there? How important are idiosyncratic shocks to
interest rates relative to the response to news that affects all rates? How is the idiosyncratic component of rates affected by time aggregation? Can one of the factors that interest rate respond to be identified as a monetary policy shock?

We analyze 21 U.S. interest rates on debt instruments with varying degrees of default risk and maturities ranging from overnight to 20 years at the daily frequency. Because we are interested in identifying common factors associated with news that affects the market, we analyze changes in the daily rates on the assumption that the daily change in interest rates is the best measure of their response to news. Despite the fact that we make no specific allowance for differences in default risk or term to maturity, we find that two factors account for most of the variation in these rates. This finding is robust to both the sample period and time aggregation. Moreover, each rate has an important idiosyncratic component whose relative importance, however, declines as the frequency of the observations is reduced; and nearly vanishes when rates are observed at the monthly frequency.

The remainder of the paper is organized as follows. Section 2 presents the data. Section 3 presents the results obtained from applying dynamic factor analysis to the interest rates described in Section 2. This section presents the analysis of alternative specifications and several robustness checks. Section 4 presents the results obtained by applying the factor analysis to weekly and monthly data. Some implications of this analysis are drawn. Section 5 investigates whether either of the factors identified in the previous sections can be attributed to monetary policy actions of the Fed. The conclusions and program for further research are presented in Section 6.

2 The Data

Our data consists of daily observations on 21 interest rates ranging in maturity from overnight to 20 years and covers the period, January 2, 1974 through August 15, 2001. There are rates on 1- and 3-month financial and nonfinancial commercial paper, fcp1, fcp3, nfcp1, and nfcp3; the 1-, 3-, 5-, 7-, 10-, and 20-year constant maturity U.S. Treasury yields, t1 through t20; the 3-, 6- and 12-month rate on U.S. Treasury bills in the secondary market, tb3, tb6 and tb12; the London bid rate on 1-, 3-, and 6-month Eurodollar deposits, ed1, ed3, and ed6, the secondary market rate on 1-, 3-, and 6-month negotiable certificates of deposit, cd1, cd3, and cd6; the effec-
tive federal funds rate, \( ffr \); and the overnight rate on repurchase agreements secured with Treasury obligations, \( rp \). The U.S. Treasury rates are free of default risks. Other rates, such as \( ffr \), are completely unsecured and, hence, may reflect a significant risk premium.

Figure 2.1 *Interest rates: Levels*

![Graph showing interest rates levels from January 2, 1974 to October 15, 2001.](image)

*Daily figures: From January 2, 1974 to October 15, 2001.*

Figure 2.1 presents all 21 interest rates over the entire sample period. While the rates clearly differ from one another, it is unquestionably the case that they share many of the same characteristics. Hence, it is not surprising that the much of the variance of the levels can be accounted for by a few common factors (see e.g., Litterman and Scheinkman, 1991). The same graph for first differences would show important short-run differences among different rates. Table 2.1 presents the correlations between first differences of all pairs of the 21 rates considered. The rates are arranged by maturity from overnight to 20 years. The correlation between the funds rate and alternative rates declines nearly monotonically as the term to maturity lengthens, and the correlation between the first-difference of the funds rate and the longer-term Treasury rates is very low. Generally speaking correlation is higher between rates of similar maturity. The correlation is also generally higher among similar assets of different maturities. This is particularly true
for Treasury rates. For example, the correlation between $tb3$ and any of the Treasury rates is higher than any of the other 3-month rates. This suggests that there may be market-specific news that affects rates in a particular market, but not other markets.\(^1\)

### Table 2.1. Correlation between rates: first differences.

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\(^1\)There are very large daily spikes in $ffr$ that occur from time to time. We undertook our analysis by deleting and not deleting these spikes. The results presented here were relatively unaffected by these observations, so these spikes are not deleted for our analysis. Sarno, Thornton and Wen (2002), who have used a subset of our data set, also report obtaining qualitatively identical results when such spikes were are were not deleted.
3 Dynamic Factor Analysis Using Daily Data

3.1 The generalized dynamic factor model

The dynamic factor model used here is summarized in this section. The interested reader will find details in FHLR. Consider a dataset consisting of \( n \) time series, each a realization of a stationary process, and assume that the following representation holds

\[
x_{it} = A_{i1}(L)u_{1t} + \cdots + A_{iq}(L)u_{qt} + \xi_{it},
\]

for \( i = 1, 2, \ldots, n \), where \( U_t = (u_{1t} \cdots u_{qt}) \) is an orthonormal white-noise vector, i.e. \( u_{jt} \) has unit variance and is orthogonal to \( u_{st} \) for any \( j \neq s \). \( A_{ij}(L) = \sum_{k=0}^{\infty} a_{ij,k} L^k \) is a polynomial (finite or infinite) in the lag operator \( L \), whose coefficients represent the impulse response function of \( x_{it} \) to the shock \( u_{jt} \). The polynomials \( A_{ij}(L) \) fulfill \( \sum_{k=0}^{\infty} a_{ij,k}^2 < \infty \). Obviously the stationarity assumption on the \( x \)'s does not rule out data sets containing non-stationary series, as stationarity can be induced either by deterministic detrending or by differencing, according to their generating process.

The shocks \( U_t \) and the components \( \chi_{it} = A_{i1}(L)u_{1t} + \cdots + A_{iq}u_{qt} \) are referred to as common shocks, common factors, or common components, respectively. We interpret the common shocks \( u_{1t}, u_{2t}, \ldots u_{qt} \) as macroeconomic shocks that affect all the variables \( x_{it} \), e.g. a demand shock, a technology shock, a monetary policy shock, etc. While the shocks are unique, each of the variables response can be different, and is represented by the polynomials \( A_{ij}(L) \).

Some of the assumptions below are formulated for \( n \to \infty \). Moreover, all estimation results are obtained asymptotically as both \( n \) and \( T \), the number of observations in each series, tend to infinity. Conceptually, our dataset is assumed to be embedded in a doubly infinite panel. In empirical situations, however, both \( n \) and \( T \) are finite, so that the reliability of our results requires that the number of series in the dataset be fairly large.

To better understand what is meant by common factors, suppose that \( q = 2 \), and that \( A_{11}(L) = c_i(L)A_{11}(L) \) and \( A_{12}(L) = c_i(L)A_{12}(L) \) for all \( i > 1 \). In this case model (1) could be rewritten as follows:

\[
\chi_{1t} = A_{11}(L)u_{1t} + A_{12}(L)u_{2t} = B_1(L)v_t
\]
\[
\chi_{it} = c_i(L)[A_{11}(L)u_{1t} + A_{12}(L)u_{2t}] = c_i(L)B_1(L)v_t = B_i(L)v_t \quad \text{for } i > 1
\]
In other words, the first and second factor would collapse into one composite factor, so that $u_{1t}$ and $u_{2t}$ could not be identified and estimated. Hence, if two or more macroeconomic factors affects all interest rates in precisely the same way, it would be impossible to distinguish between them. Thus, in order for the number $q$ in (1) to make sense, it is important that each variable be permitted to respond differently to the common shocks, i.e. that the polynomials $A_{ij}(L)$ are sufficiently different across different variables.

The components $\xi_{it}$ in equation (1) are referred to as the idiosyncratic components. We suppose that $\xi_{it}$ is orthogonal to all components of $U_t$ at any lead and lag, and therefore orthogonal to $\chi_{st}$ at any lead and lag. The usual additional assumption is that the idiosyncratic components $\xi_{it}$ are mutually orthogonal at any lead and lag, i.e. that $\xi_{it}$ contains information that is specific only to $x_{it}$. Here, however, following FHLR we use a weaker assumption, whose introduction requires the spectral density matrix of the vector $(x_{1t} \ x_{2t} \ \cdots \ x_{nt})$, which is denoted by $\Sigma_n(\lambda)$. Orthogonality between the $\xi$’s and the $\chi$’s implies that

$$\Sigma_n(\lambda) = \Sigma_n^\chi(\lambda) + \Sigma_n^\xi(\lambda),$$

where $\Sigma_n^\chi(\lambda)$ and $\Sigma_n^\xi(\lambda)$ denote the spectral density matrices of the vectors of common and idiosyncratic components respectively. Denote the $j$-th eigenvalues (in descending order) of $\Sigma_n(\lambda)$, $\Sigma_n^\chi(\lambda)$ and $\Sigma_n^\xi(\lambda)$ by $\lambda_{nj}(\lambda)$, $\lambda_{nj}^\chi(\lambda)$, $\lambda_{nj}^\xi(\lambda)$, respectively.

**Assumption 1.** For $n \to \infty$ and $j \leq q$, the eigenvalues $\lambda_{nj}^\chi(\lambda) \to \infty$ for almost any $\lambda \in [-\pi \ \pi]$.

**Assumption 2.** There exists a positive real $\Lambda$ such that $\lambda_{n1}^\xi(\lambda) \leq \Lambda$ for any $n$.

Assumption 2 is a generalization of the case in which the idiosyncratic components are mutually orthogonal and their variance is bounded with respect to $n$ (this is why the model is called a *generalized* dynamic factor model). Thus, some limited covariance is not ruled out for the idiosyncratic components. Assumption 1 guarantees that the factors $u_{jt}$ do not collapse into a smaller number of factors. Forni and Lippi (2001) prove that Assumption 1 and 2 are equivalent to the following assumption, which is formulated in terms of the “observable” matrix $\Sigma_n(\lambda)$.

**Assumption 3.** For $n \to \infty$ and $j \leq q$, the eigenvalues $\lambda_{nj}(\lambda) \to \infty$ for almost any $\lambda \in [-\pi \ \pi]$. There exists a positive real $M$ such that $\lambda_{n,q+1}(\lambda) \leq M$ for any $n$. 

7
Assumption 3 can be used as a basis for a heuristic criterion to select the number of factors. Given the number \( n \) of variables in our dataset, we can compute the spectral matrices and the corresponding eigenvalues for each \( m \leq n \). The number \( q \) should correspond to the number of clearly diverging eigenvalues.

**Figure 3.1** Eigenvalues as functions of the number of variables

The graphs in the first row, from the left, correspond to the frequencies 0, \( \pi/3 \), in the second row to \( 2\pi/3, \pi \).

Under Assumption 3, and additional technical assumptions, FHLR construct a consistent estimator for the components \( \chi_{it} \) and \( \xi_{it} \) based on the dynamic principal components. This will not be discussed here. It is important to point out, however, that estimation of the common components does not imply identification of the common shocks. In other words, once the \( \chi \)'s have been consistently estimated, there exist an infinite number of representations, like the one in (1),

\[
\chi_{it} = \sum_{j=1}^{q} B_{ij}(L)v_{jt},
\]

where \( V_t = (v_{1t} \ v_{2t} \ \cdots \ v_{qt}) \) is an orthonormal white-noise vector linked to the structural vector \( U_t \) by \( V_t = SU_t \), \( S \) being a unitary matrix. Identification of \( U_t \) among all possible vectors of shocks requires restrictions, just like
identification of SVAR models. The advantage of the dynamic factor model (1) approach is that the number of shocks does not increase with the number of variables. In contrast, in Structural VAR (SVAR) models the restrictions required to achieve identification increases as the square of the number of variables included in the VAR.

Figure 3.2 Variance-ratio of idiosyncratic component to the variable

The four lines, from top to bottom, correspond to the frequency band $[0 \rho]$, with $\rho$ taking the values $\pi, \pi/4, \pi/16, \pi/32$. The variables, indicated in this Figure, as well as in Figures 4.1, 5.1, 5.2, by numbers from 1 to 21 are fcp1, fcp3, nfcp1, nfcp3, t1, t3, t5, t7, t10, t20, tb3, tb6, tb12, ed1, ed3, ed6, cd1, cd3, cd6, ffr, rp.

3.2 Dynamic Factor Analysis of the Interest Rates

The asymptotic results in FHLR do not change if the variables are rescaled. In particular, estimation of $\chi_{it}$ is not affected by normalization (each variable divided by its standard deviation) as $n$ tends to infinity. When $n$ is finite, however, the relative variances of the variables $x_{it}$ may matter. This is especially true when $n$ is relatively small, as in our case. Consequently, all of the interest rates have been normalized. Figure 3.1 plots the eigenvalues of the spectral density matrix of the vector $(\Delta R_{1t} \Delta R_{2t} \cdots \Delta R_{mt})$, for $m = 1, 2, \ldots, 21$ (obviously the $s$-th eigenvalue exists only from $m = s$ onward), for the frequencies 0, $\pi/3$, $2\pi/3$ and $\pi$. The first two eigenvalues stand out and explain a large proportion of the total variance. Indeed, at frequencies 0 and $\pi$ the contributions of eigenvalues following the second are essentially nil. At the central frequencies, there is some contribution for the eigenvalues following the second; however, the contribution is relatively
small. Consequently, based on Assumption 3, we estimate a dynamic factor model with $q = 2$.\footnote{The exercises presented below in Sections 3.2, 4 and 5, based on the two-factor model, have been replicated within a three-factor model, with no qualitative changes for the results.}

Figure 3.2 presents the variance ratio $\frac{\text{var}(\xi_{it})}{\text{var}(\Delta R_{it})}$ for all 21 interest rates for the frequency bands from zero to $\pi$ (all periods), $\pi/4$ (periods of 8 days or longer), $\pi/16$ (32 days or longer), $\pi/32$ (64 days or longer) respectively. The integer numbers on the horizontal axis represent the interest rates from 1 to 21. Figure 3.2 shows that the contribution of the common components to the variance of each of the interest rates is far more important at lower frequencies. With the exception of $\text{ffr}$ and $\text{rp}$, the ratio falls beneath 0.1 when the band $[0, \pi/32]$ is considered. Consistent with Duffee (1996) the variance of the idiosyncratic component of the 3-month t-bill rate also remains somewhat large at low frequencies. In any event, two factors appear to explain much of the variation in the 21 interest rates at low frequencies and are sufficient to explain nearly all of the long-run variation in rates. These results imply that when the data are aggregated over time, the variance ratio of idiosyncratic components for each rate will become smaller. Indeed, as we see in Section 4, the idiosyncratic component of rates almost disappears with monthly data.

The spectral density of the idiosyncratic components, at each frequency, for $\text{nfcpl}$, $\text{edl}$, $\text{t10}$, and $\text{ffr}$ are graphed in Figure 3.3, together with the corresponding total spectral densities (sum of the common and idiosyncratic spectral densities). The idiosyncratic spectral density increases slightly for $\text{nfcpl}$, $\text{edl}$ and $\text{t10}$, but increase substantially for $\text{ffr}$ (and for $\text{rp}$). In contrast, the corresponding total spectral densities are either downward sloping, or in the case of $\text{ffr}$, increase less than the idiosyncratic component. Interestingly, the spectral density of the $\text{ffr}$ has three outstanding peaks that match quite closely those obtained by Sarno, Thornton and Wen (2002) using a different (parametric) estimation technique.\footnote{It should be noted, however, that the “idiosyncratic” component identified by Sarno, Thornton and Wen (2002) differs from the one estimated here.}

It is interesting to note that spectral densities of the idiosyncratic components of all the rates is nearly zero at the zero frequency. This is suggests that all of the idiosyncratic shocks are temporary, having no permanent effect on individual rates and, consequently, the structure of rates. While there is nothing that requires all idiosyncratic shocks to be temporary, we are
clined to interpret this as evidence that our DFM has correctly identified the idiosyncratic component of rates.

**Figure 3.3** *Spectral density of the variable and the idiosyncratic component*

![Graphs](image)

The variables in the first row, from the left, are nfcp1 and ed1, respectively. The second row reports t10 and ffr, respectively. Cycles per day on the horizontal axis.

The results suggested by Figure 3.2 are confirmed for all but the overnight rates. For the overnight rates, the relative variance of the idiosyncratic component increases rather than decreases as the frequency is reduced. The relative variance of the idiosyncratic component of the overnight rates is more than 45 percent at frequency $\frac{\pi}{32}$, and remains relatively larger even at frequencies very near zero. Hence, it appears that the idiosyncratic component of the overnight rates has become both larger and more persistent (essentially permanent) since 1994. Finally, the spectral density of $ffr$ has lost its three peaks.
4 Dynamic Factor Analysis Results Using Weekly and Monthly Data

To further investigate the effects of time aggregation, the analysis in the previous section is repeated using weekly and monthly average data. Time aggregation *per se* does not affect results on the number of factors. An analysis equivalent to that reported in the previous sections indicated that there are two common factors. Given the result in the previous section that high frequency cycles contribute very little to the spectral density, we expect that this will hold for time-aggregated data as well.

Moreover, we expect that this result will occur for both period-average and end-of-period data. To understand why, let $x_t$ be I(1), where $t$ indicates days. Stationarity is achieved by taking first differences $x_t - x_{t-1}$. Suppose that one wants to sample weekly. This can be done in two steps: (1) fifth differencing, i.e. taking $x_t - x_{t-5}$, for which we have

$$x_t - x_{t-5} = (x_t - x_{t-1}) + \cdots + (x_{t-4} - x_{t-5}) = (1 + L + L^2 + L^3 + L^4)(1 - L)x_t,$$

and (2) sampling the fifth difference at times 0, 5, 10, ... As is well known, the gain of the filter $(1 + L + L^2 + L^3 + L^4)$ used in step (1) is mainly concentrated in the low-frequency band, so that sampling weekly entails a considerable smoothing. An even more serious smoothing effect arises if we aggregate over time. In that case we firstly take $(1 + L + L^2 + L^3 + L^4)x_t$, then take the fifth difference and then sample, so that the final filter is $(1 + L + L^2 + L^3 + L^4)^2(1 - L)$. Step 2, i.e. sampling, has a less clear effect; however, in no case does it offset the effect of the smoothing filters.

To investigate the effects of time aggregation on the relative importance of the idiosyncratic components of our dynamic factor analysis, we repeated the exercise conducted in Section 3.2 using weekly and monthly observations, with both period-average and end-of-period data. As expected, the analysis again strongly suggests that there are but two common factors.

Figure 4.1 reports the variance-ratio of the idiosyncratic component (over the frequency band $[0 \pi]$ for all 21 interest rates using daily data, and for weekly and monthly period-average data. As expected, the relative importance of the idiosyncratic component declines substantially as the horizon of the time aggregation increases. Indeed, with the exceptions of $ffr$ and $rp$, the contribution relative variance of idiosyncratic component for monthly period-average data is very similar to that measured at frequency $\pi/32$. For $ffr$ and $rp$, the relative variances for monthly data are nearly twice that measured
at frequency $\pi/32$. As expected, the results with end-of-period observations are essentially the same and, consequently, not reported.

**Figure 4.1** Variance-ratio of idiosyncratic component to the variable

The three lines, from top to bottom, correspond to daily, weekly, monthly figures respectively. The top line is identical to the top line in Figure 3.2.

These results suggest that each market interest has a significant idiosyncratic component when measured at the daily frequency. Since, by construction, the idiosyncratic component of each rate is nearly orthogonal to that of the other rates, it reflects market specific shocks. This is consistent with the fact that the idiosyncratic components are relatively large for the overnight rates ($ffr$ and $rp$), the Eurodollar deposit rates (all three maturities) and both of the commercial paper rates. The relative importance of the idiosyncratic component is relatively small for the Treasury rates with the exception of $tb3$, which is consistent with Duffee’s (1996) finding. For all rates, however, the relative importance of the idiosyncratic component declines dramatically with time aggregation.

5 Does One of the Factors Represent a Monetary Policy Shock?

The fact that the response of interest rates to news can be summarized by a few common components has important implications for macroeconomic analyses. For example, in SVAR literature the variations in short-term interest rates that cannot be accounted for by the past and contemporaneous behavior of economic variables that precede $ffr$ in the VAR, are assumed
to reflect the effect of exogenous monetary policy actions on interests via the “liquidity effect.” If the news that interest rates respond to—news about monetary policy, industrial production, consumer confidence, inflation, the list goes on and on—is concentrated in a few common factors, however, separating the effect of one source of news from others will be complicated.

Hamilton (1997) has criticized the recursive SVAR (RSVAR) approach to identifying monetary policy shocks, arguing that, because policy actions are frequently the response of the Fed to new information about the economy, “The correlation between such a “policy innovation” and the future level of output of necessity mixes together the effect of policy on output with the effect of output forecasts on policy.” He suggests that the identification of an exogenous policy action is best measured using daily data.

Because the response to rates to “new information” is represented by a few common factors, the identification of monetary policy shocks will be difficult even with daily data. Nevertheless, we attempt to identify monetary policy shocks with our DFM. As with SVAR’s, identification in DFM requires identifying restrictions. Like Hamilton (1997) and Thornton (2001) we use daily data and rely on aspects of the Fed’s operation procedure to identify monetary policy. The approach is novel in that it relies on the fact that monetary policy actions that are unknown to the public should initially affect only the federal funds rate. Other interest rates will change only when the market is aware of a persistent change in the funds rate.

The Trading Desk of the Federal Reserve Bank of New York (hereafter, desk), carries out open market operations with the expressed purpose of target the federal funds rate at the level set by the Federal Open Market Committee (FOMC). Since 1994 the FOMC has announced target change upon the decision to change the target. Hence, it is now the case that the funds rate and other rates change upon the announcement. This means that the desk need not immediately engage in open market operations in order to affect the federal funds rate and other short-term rates (e.g., Taylor, 2001). More importantly, this means that, since 1994, monetary policy actions should be no more reflected in the federal funds rate than in other short-term rates. Prior to 1994, however, the Fed did not announce changes in the funds rate target. Moreover, Poole, Rasche and Thornton (2002) show that there were only a few occasions when the market knew that the target had changed on the day that the action was taken. Hence, as a general rule, monetary policy actions

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(i.e. desk open market operations designed to change the level of the funds rate) should have impacted the funds rate immediately. Hence, if one of the two macroeconomic shocks is a monetary policy shock, its impact should be reflected immediately in the ffr and only subsequently in other rates. In contrast, after 1994 all the rates should be affected contemporaneously by announcements of funds rate target changes.

To understand how we operationalize our identification procedure, we respecify model (1) with \( n = 21 \) and \( q = 2 \),

\[
x_{it} = \chi_{it} + \xi_{it} = A_{i1}(L)u_{1t} + A_{i2}(L)u_{2t} + \xi_{it}
= a_{i1,0}u_{1t} + a_{i1,1}u_{1t-1} + \cdots + a_{i2,0}u_{2t} + a_{i2,1}u_{2t-1} + \cdots + \xi_{it}
\]

\[ (2) \]

If \( u_{1t} \) is identified as the shock to monetary policy, and we assume that ffr is impacted before the other rates, then the following restrictions hold

\[
a_{i1,s} = 0 \quad \text{for all } i \neq 20.
\]

\[ (3) \]

Since \((u_{1t} \ u_{2t})\) is an orthonormal white noise, orthogonal to the idiosyncratic components, the restrictions given by (3) are equivalent to

\[
\text{cov}(u_{1t}, x_{it}) = \text{cov}(u_{1t}, \chi_{it}) = 0 \quad \text{for all } i \neq 20.
\]

\[ (4) \]

Hence, the restrictions given by (3) are nothing other than the usual recursive identification restriction that require one variable to be affected before the others by the shock. The SVAR model would be just identified with two shocks and two variables. However, our model contains two shocks and 21 variables, so that with (3) the model is overidentified.

We do not undertake a formal analysis of these overidentifying restrictions. Rather, we simply investigate the likelihood that these restrictions will hold. To see how this is done, consider two of the common factors, the one for ffr and the \( j \)-th, with \( j \neq ffr \). From (2):

\[
\chi_{fr,t} = A_{fr,1}(L)u_{1t} + A_{fr,2}(L)u_{2t}
\chi_{jt} = A_{j1}(L)u_{1t} + A_{j2}(L)u_{2t}
\]

\[ (5) \]

Suppose now that the matrix

\[
B_j(L) = \begin{pmatrix}
A_{fr,1}(L) & A_{fr,2}(L) \\
A_{j1}(L) & A_{j2}(L)
\end{pmatrix}
\]
is invertible\(^5\) and let

\[
C_j(L) = B_j(L)^{-1} = \begin{pmatrix}
C_{ffr,1}(L) & C_{ffr,2}(L) \\
C_{j1}(L) & C_{j2}(L)
\end{pmatrix}.
\]

We may rewrite (5) as

\[
\begin{align*}
C_{ffr,1}(L)\chi_{ffr,t} &= -C_{ffr,2}(L)\chi_{jt} + u_{1t} \\
C_{j2}(L)\chi_{jt} &= -C_{j1}(L)\chi_{ffr,t} + u_{2t}.
\end{align*}
\]

By an obvious change in notation, the first equation in (6) can be rewritten as

\[
\chi_{ffr,t} = \alpha_1\chi_{ffr,t-1} + \alpha_2\chi_{ffr,t-2} + \cdots + \beta_0\chi_{jt} + \beta_1\chi_{jt-1}\beta_2\chi_{jt-2} + \cdots + v_t,
\]

where \(v_t = \gamma u_{1t}\), \(\gamma\) being the reciprocal of \(c_{ffr,1,0}\).

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\(^5\)For a thorough discussion of this point see Forni, Lippi and Reichlin (2002).
The restrictions (3) imply that $v_t$ is orthogonal to $\chi_{jt}$. Thus (7) is the projection of $\chi_{fr,t}$ on $\chi_{jt}$, past values of $\chi_{fr,t}$ and past values of $\chi_{jt}$. Moreover, the residual of (7) should be independent of $j$, up to a multiplicative constant.

Regression (7) has been run for all possible values of $j$. The correlation of the residual $v_t$ with $\chi_{kt}$, $k = ffr, rp, ..., T20$, is $\sigma_v^2$ for $k = ffr$, approximately zero for $k = j$ by construction, and should be approximately zero under restriction (3) for $k \neq ffr$ and $k \neq j$. Figure 5.1 shows the correlations between $v_t$ and $\chi_{kt}$ when (7) is estimated with $j = t10$. The shape of the figure is fairly typical of those obtained using other rates as the $j$-th rate, with some correlations low but others rather high. Moreover, essentially the same results are obtained using weekly and monthly data.

![Figure 5.2](image)

**Figure 5.2** Correlation between $v_t$, obtained as the residual in (7) with $j = 9$, and $\chi_{kt}$, $k = 1, 2, \ldots, 21$. Period 1985-1993, solid line. Period 1994-2001, dashed line.

While not conclusive, the size of the correlations in 5.1 strongly suggest that the restrictions in 3 are unlikely to hold. To further investigate the strength of these results, the same analysis is applied to data after 1993. Because the Fed announced funds rate target changes after 1994, we should expect to find a marked difference in the information initially reflected in ffr between the periods 1985-1993 and 1994-2001. In Figure 5.2 we report
the same correlations as in Figure 5.1 for the period 1985-1993, solid line, 1994-2001, dashed line. The second subsample corresponds to the period in which the FOMC announces target changes. There is little change in the pattern of correlations. If anything, some of the correlations for the 1994-2001 period are lower than those for the 1985-1993 period, suggesting that there was a “increase of uniqueness” for the funds rate after the Fed began announcing target changes. The rise in the uniqueness of $ffr$ does not appear to be particularly large, however. In any event, evidence suggests that $ffr$ did not uniquely reflect monetary policy shocks before 1994.\textsuperscript{6} Moreover, there is little evidence of a marked change in the contemporaneous information reflected in interest rates at the daily frequency in response to the dramatic changes in the FOMC’s procedures. Given the evidence that the idiosyncratic components of rates declines markedly when the data are time aggregated, the results should be even be less compelling using weekly or monthly data.

6 Conclusions

This papers uses a DFM to analysis the information content of news that affects market interest rates. We find that, while market rates are buffeted by news from a variety of sources, the response of rates to news appears to be represented by two common factors. Because interest rates are thought to have both real and expected inflation components, it is natural to think that one of these factors represents the ”real” component of rates while the other represents the ”inflation expectations” component. This cannot be established without additional identifying assumptions, however, which is beyond to scope of the present analysis.

The fact that information from a variety of sources appears to be reflected in a few common factors has implications for researcher’s ability to identify specific sources of shocks to interest rates, e.g., monetary policy. This problem is likely to be more severe the higher the degree of temporal aggregation for two reasons. First, because the information that market interest rates respond to occurs at relatively high frequencies, say daily, distinguishing between the response of alternative sources of news requires high frequency data. It is well known that time aggregation can distort the effect of high

\textsuperscript{6}This is consistent with the evidence provide by Garfinkel and Thornton (1995) and Sarno, Thornton, and Wen (2002) who show that information in the funds rate is not unique.
frequency information. Hence, attributing a “shock” identified at the monthly or quarterly frequencies to a specific high-frequency source is problematic at best.

Second, reducing the frequency of the data also increases the likelihood that interest rates reflect information that is not publicly known or announced. Private information, that causes one rate to change relative to other rates, creates arbitrage opportunities. As market participants exploit these opportunities, the effect of private information—that was initially reflected in one rate—will propagate to other market rates. Generally speaking, the longer the period of time over which interest rates are measured, the more likely it is that information that was initially reflected only in one rate affects other rates.

Our analysis also shows that each rate analyzed has an important idiosyncratic source of news that decreases with the level of temporal aggregation, and nearly vanishes at the monthly frequency. This suggests that while market specific information plays a role in the variability of interest rates at the daily frequency, such information is much less responsible for the variance of rates at lower frequencies. Moreover, the fact that the spectral densities of the idiosyncratic component of all rates at the zero frequency is essentially zero suggests that idiosyncratic shocks to rates have no permanent effect on the structure of rates.

Finally, we attempt to identify a monetary policy shocks to interest rates at the daily frequency by using the fact that policy actions that are unknown to the public should effect the funds rates contemporaneously and only subsequently other rates. Unfortunately, we were unable to identify a unique monetary policy shock. This is disconcerting because the effect of open market operations should be reflected initially in the federal funds market. Nevertheless, this result is consistent with other attempts at identifying monetary policy shocks using daily data (e.g., Thornton, 2001, 2003).

7 References